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Homework #84

CSc 245 Section Z

SL: Glaeken Trismegestus

1. Section 1.1 #27(a):

   Converse: If I ski tomorrow, it snowed today.

   Contrapositive: If I do not ski tomorrow, it did not snow today.

   Inverse: If it does not snow today, I will not ski tomorrow.

2. Section 1.1 #31(c):

p	q	$\neg q$	$p \vee \neg q$	$(p \vee \neg q) \to q$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

3. Section 1.3 #19: Answering two ways, neither by truth table:

   (a) By a sequence of logical equivalances:

$$\begin{aligned}
\neg p \leftrightarrow q &\equiv (\neg p \to q) \wedge (q \to \neg p) && [\textit{Def. of Biimplication}]\\
&\equiv (\neg q \to \neg\neg p) \wedge (\neg\neg p \to \neg q) && [\textit{Contrapositive } (\times\ 2)]\\
&\equiv (\neg q \to p) \wedge (p \to \neg q) && [\textit{Double Negation } (\times\ 2)]\\
&\equiv (p \to \neg q) \wedge (\neg q \to p) && [\textit{Commutativity } (\times\ 2)]\\
&\equiv p \leftrightarrow \neg q && [\textit{Def. of Biimplication}]
\end{aligned}$$

   (b) By reasoning: We know that $p \leftrightarrow q$ is equivalent to $(p \to q) \wedge (q \to p)$, and that $p \leftrightarrow q$ is true only when p and q are equal (that is, either both true or both false).

   When we negate p, the cases in which p and q were equal are now cases in which they differ, and vice versa. This makes $\neg p \leftrightarrow q$ the negation of $p \leftrightarrow q$.

   Negating q forces the same swapping of cases, and so has exactly the same result. Thus, $\neg p \leftrightarrow q \equiv p \leftrightarrow \neg q$.

4. Section 1.4 #9:

   (b) $\exists x(P(x) \wedge \neg Q(x)), x \in$ UA Students

   (c) $\forall x(P(x) \vee Q(x)), x \in$ UA Students

5. Section 1.7 #1:

   Conjecture: The sum of two odd integers is even.

   Proof (Direct): Assume that $a, b \in \mathbb{Z}^{\text{odd}}$.

   Any odd integer is one more than some even integer. Let $a = 2k + 1$ and $b = 2j + 1$, where $k, j \in \mathbb{Z}$. It follows that $a + b = (2k + 1) + (2j + 1) = 2k + 2j + 2 = 2(k + j + 1)$. This result shows that $a + b$ is twice some integer, which is the definition of an even number.

   Therefore, the sum of two odd integers is even.

6. Section 2.2 #5:

   Conjecture: $\overline{\overline{A}} = A$.

   Proof (Direct):

   Case 1: Show that $\overline{\overline{A}} \subseteq A$ is true.

$$\begin{aligned}
\overline{\overline{A}} \subseteq A &\equiv \forall z\,(z \in \overline{\overline{A}} \rightarrow z \in A) && [\textit{Def. of } \subseteq] \\
&\equiv \forall z\,(\overline{z \in \overline{A}} \rightarrow z \in A) && [\textit{Def. of } \overline{X}] \\
&\equiv \forall z\,(\overline{\overline{z \in A}} \rightarrow z \in A) && [\,''\,] \\
&\equiv \forall z\,(z \in A \rightarrow z \in A) && [\textit{Double Negation}] \\
&\equiv \forall z\,(\mathbf{T}) && [\textit{Self-Implication}] \\
&\equiv \mathbf{T} && [\textit{Tautology}]
\end{aligned}$$

   Case 2: Show that $A \subseteq \overline{\overline{A}}$ is true.

   (*This half is very similar to the above, and so is not provided here.*)

   Therefore, $\overline{\overline{A}} = A$.

7. Section 2.2 #13:

   Conjecture: $A \cap (A \cup B) = A$.

   Proof (Direct):

$$\begin{aligned}
A \cap (A \cup B) &= (A \cup \emptyset) \cap (A \cup B)) && [\textit{Identity}] \\
&= A \cup (\emptyset \cap B) && [\textit{Distribution}] \\
&= A \cup \emptyset && [\textit{Domination}] \\
&= A && [\textit{Identity}]
\end{aligned}$$

   Therefore, $A \cap (A \cup B) = A$.

8. Section 2.6 #3(a):

$$AB = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 4 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 11 \\ 2 & 18 \end{bmatrix}$$

9. Section 5.1 #9:

   (a) $1, 2, \ldots, n$ is an arithmetic sequence. The sequence of sums of the terms ($s_1 = 1, s_2 = 3, s_3 = 6, s_4 = 10, s_5 = 15, \ldots$) is an arithmetic series.

   There are multiple ways of figuring this out. If we look at the values in the series and use some imagination to try to express s_n in terms of n, we (eventually) discover that s_n is half of the product $n(n+1)$.

   Here's another way: In general, the sum of the n terms of an arithmetic sequence a is $s_n = \frac{n(a_1+a_n)}{2}$. In this problem, $a_1 = 1$ and $a_n = n$. Substituting, our sum's formula is $s_n = \frac{n(n+1)}{2}$. Again, that's half of the product $n(n+1)$.

   (b) Conjecture: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

   Proof (by Weak Induction):

   Basis Step:

   Let $n = 1$. $\sum_{i=1}^{1} i = 1 = \frac{1(1+1)}{2}$. OK!

   Inductive Step:

   If $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$, then $\sum_{i=1}^{n+1} i = \frac{(n+1)(n+2)}{2}$.

$$\begin{aligned} \sum_{i=1}^{n+1} i &= \sum_{i=1}^{n} i + (n+1) && [\textit{Expose the I.H.'s sum}] \\ &= \frac{n(n+1)}{2} + (n+1) && [\textit{By the I.H.}] \\ &= \frac{n^2+n+2n+2}{2} && [\textit{Algebra}] \\ &= \frac{(n+1)(n+2)}{2} && [\text{''}] \end{aligned}$$

   Thus, $\sum_{i=1}^{n+1} i = \frac{(n+1)(n+2)}{2}$.

   Therefore, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

Some Additional Examples:

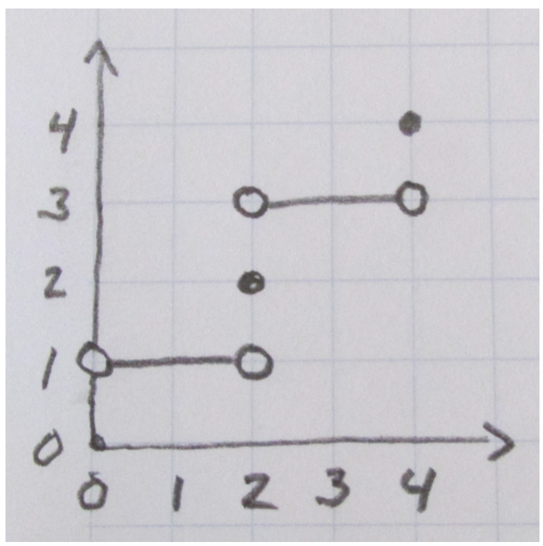
- Formatting a Piecewise Function:

$$f(n) = \begin{cases} \frac{n}{2} & \text{when } n \equiv 0 \pmod{2} \\ 3n+1 & \text{otherwise} \end{cases}$$

- Importing an Image

  Say that a homework answer requires a diagram to be produced. In a more perfect world, you'd use a diagramming tool to make a crisp, impressive picture. In a world with weekly homework due dates, you might decide to save time by drawing it by hand. Either way, how do you import the picture into your LaTeX file?

  LaTeX users have created a large number of add-ons. One of the more commonly-used packages is `graphicx`. When used with `pdflatex`, you can import `JPG` and `PNG` images easily. For example:



  Many image processing programs (e.g., GIMP) can convert images into `JPG` or `PNG` formats. Strong suggestion: Reduce the resolution of the image while you're at it. Your cell phone might capture 12 megapixel pictures, but importing one of those will make your `PDF` file enormous. The `PNG` image above is just 500 by 500 pixels in size (a mere 0.25 megapixels and just 325K bytes – less than a tenth of the size of the original), and is plenty large and entirely legible.